Task-Level Fiscal Policy for the Green Transition:
Moving Beyond Binary Classifications

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ABSTRACT

This paper develops a task-based general-equilibrium model to evaluate targeted green-input subsidies, offering new insights into fiscal policy for the green transition. Production is represented as a continuum of tasks ranked by "greenness," moving beyond the conventional "green vs. dirty" sectoral split. By capturing task-level heterogeneity, the model shows that competitive markets allocate too few tasks to green inputs relative to a planner who internalizes environmental externalities, justifying corrective subsidies. The framework addresses three policy-relevant questions: (i) *Design*—how should subsidies vary across tasks to maximize reallocation toward green methods? (ii) *Trade-offs*—what productivity cost, if any, accompanies environmental gains? (iii) *Financing*—which tax base funds subsidies with the least distortion? Key findings are: (1) subsidies work best when the productivity gap between green and traditional inputs is small, (2) in a U.S. calibration, welfare improves only if the externality parameter ψ is roughly three times the standard public-goods benchmark (1.2 vs. 0.4), implying a 4.3 % consumption-equivalent threshold, and (3) lump-sum taxes impose the smallest welfare loss, followed by capital taxes, while labor-income taxes are most distortionary. By integrating fiscal policy design with macro-economic outcomes, this task-level approach provides a more realistic foundation for climate policy, guiding interventions that align environmental goals with macroeconomic efficiency.

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1 Introduction

Mitigating climate change requires rethinking how we produce goods and use energy. To meet the Paris Agreement's 1.5°C target, global emissions must fall by 45% by 2030 and reach net-zero by 2050 (United Nations, 2023). In response, governments have adopted large-scale policy packages—such as the U.S. Inflation Reduction Act, Canada's Sustainable Jobs Plan, and the EU Green Deal—to achieve carbon neutrality by mid-century. A central tool in these packages is targeted subsidies that shift investment toward cleaner, more sustainable production (IRA, 2023). Implementing these subsidies effectively raises three questions: (i) *Design*—which subsidy structure maximizes impact? (ii) *Trade-offs*—what productivity cost, if any, accompanies environmental gains? and (iii) *Financing*—how can subsidies be funded with minimal distortion? This paper answers these questions using a general-equilibrium model.

Most climate–macro models split the economy into "green" and "dirty" sectors, then recommend taxing the latter to redirect resources to the former. In reality, however, production involves a continuum of tasks with varying environmental footprints. For example, building an electric vehicle involves tasks from mineral extraction to final assembly, each with distinct impacts. A binary sectoral classification obscures this complexity and can misguide policy design. To address this gap, I decompose production into a continuum of tasks, each indexed by its "greenness," and allow each task to be performed with either green or traditional inputs.

I build on the framework of Acemoglu and Restrepo (2018) from the automation literature, but introduce three key changes. First, I replace their complexity index with a *greenness index* (Vona, Marin, Consoli, and Popp, 2018a) that measures each task's environmental impact. Second, I allow two types of inputs—*green* (which generate positive environmental spillovers) and *traditional* (which do not)—to capture externalities. Third, I incorporate task-level comparative advantage: green-skilled labor is relatively more productive in tasks with higher greenness scores, leading to endogenous task allocation.

This structure uncovers a market failure: competitive markets allocate too few tasks to green inputs, whereas a planner who internalizes environmental benefits would assign significantly more tasks to green methods. That misallocation motivates targeted green subsidies. Moreover, the task-level setup directly addresses the three guiding questions. Subsidies can be tailored to task productivity gaps (*Design*); the model quantifies the productivity loss versus environmental benefit for each task (*Trade-offs*); and a government budget constraint allows comparison of lump-sum, labor-income, and capital-income financing (*Financing*).

The model delivers clear answers. **First**, green-input subsidies are most effective when the productivity gap between green and traditional inputs is narrow. Policymakers should thus target sectors where green methods already approach traditional productivity and complement subsidies with R&D or infrastructure to narrow remaining gaps. **Second**, calibrated to U.S. data, a 5% green-labor subsidy raises welfare only if the environmental externality parameter ψ exceeds roughly 1.2—three times the standard public-goods benchmark—implying a 4.3% consumption-equivalent gain. The elasticity of substitution among tasks χ, moderates this threshold: higher χ lowers the required ψ by easing task reallocation. **Finally**, among lump-sum, labor-income, and capital-income taxes, lump-sum financing is least distortionary, capital taxes are second-best, and labor taxes are most costly. This ranking reverses the classic Chamley–Judd result (Chamley, 1986; Judd, 1985) once task-level externalities are introduced: taxing labor directly raises the cost of greener tasks, whereas capital taxes spread distortions more evenly.

These findings provide a rigorous, task-level framework for assessing green subsidies. By moving beyond a simple "green versus dirty" split, the model fills a key gap in climate–macro research and offers practical guidance for designing and funding climate policies that balance environmental and economic objectives.

2 Literature Review

The task-based model, developed by Acemoglu and Autor (2011), fundamentally reconceptualizes production by focusing on job tasks rather than traditional production factors. This framework, which allocates tasks based on comparative advantage, has become instrumental for studying structural economic transformations, particularly in automation research (Acemoglu and Autor (2011); Acemoglu and Restrepo (2018); Hémous and Olsen (2021)). While most studies examine automation's effects on wage distribution and labor share, applied microeconomists have adapted task-based approaches to characterize green jobs and skills using granular occupational data from sources like the Occupational Information Network (O*NET) (Consoli, Marin, Marzucchi, and Vona (2016); Vona, Marin, and Consoli (2019); Vona (2021)). Bontadini and Vona (2020) and Vona et al. (2018a) further refine these measures. Despite these empirical advances, the theoretical adaptation of task-based frameworks to evaluate green policies remains underdeveloped—a critical gap this paper addresses by incorporating environmental externalities into the task-based model.

Traditional climate-macro models primarily approach environmental externalities through taxation mechanisms (Nordhaus and Boyer (2000); Angelopoulos, Economides, and Philippopoulos (2010); Fischer and Springborn (2011); Heutel (2012); Golosov, Hassler, Krusell, and Tsyvinski (2014); Hassler, Krusell, and Jr (2016); Fried (2018); Barrage and Nordhaus (2023); Traeger (2023)). However, recent climate policies, exemplified by the U.S. Inflation Reduction Act, increasingly emphasize targeted subsidies as primary policy instruments. Newell, Pizer, and Raimi (2019) provide a comprehensive overview of green energy subsidies, noting their historical underrepresentation compared to carbon taxes. Studies such as Hassler, Krusell, Olovsson, and Reiter (2020) and Casey, Jeon, and Traeger (2023) explore subsidies' impact on green energy RD and production, finding potential increases in dirty energy use. Research by Palmer and Burtraw (2005) and Fischer and Newell (2008) suggests that, although subsidies reduce emissions in static models, they are less efficient than methods like emissions pricing. Similar conclusions are drawn in dynamic settings by Gerlagh and der Zwaan (2006) and Kalkuhl, Edenhofer, and Lessmann (2013). Benkhodja, Fromentin, and Ma (2023) compare different green subsidies and find that subsidizing labor costs of green firms is most effective in reducing pollution. Building on this literature, the paper evaluates green-input subsidies within a task-based framework, focusing on three dimensions—*design*, *trade-off*, and *financing*.

Labor market policies represent a critical yet understudied dimension of the green transition, as a skilled workforce is essential for diffusing climate-friendly technologies (Tyros, Andrews, and de Serres (2024)). While most climate-macro research prioritizes capital and technology investments, labor market interventions are equally important, particularly regarding transitional unemployment during decarbonization (Bluedorn, Hansen, Noureldin, Shibata, and Tavares (2023)). Existing research at the intersection of environmental and labor economics explores climate change's effects on labor supply and productivity (Zivin and Neidell (2012); Hsiang, Kopp, Jina, Rising, Delgado, Mohan, Rasmussen, Muir-Wood, Wilson, Oppenheimer, Larsen, and Houser (2017)). Studies also investigate the distributional effects of environmental policies, linking outcomes to changes in relative demand for capital and labor (Araar, Duclos, Rivest, and Fortin (2011); Rausch, Metcalf, Reilly, and Paltsev (2011); Fullerton, Heutel, and Metcalf (2012); Goulder, Hafstead, Kim, and Long (2019)). However, explicit examination of effective policy design to reallocate labor from brown to green jobs is a crucial step for achieving climate goals (Tyros et al. (2024)). This paper addresses this gap by proposing policies that optimize labor allocation, providing direct policy relevance. While I focus primarily on labor inputs due to the availability of occupational greenness indices from O*NET, the key insights apply to any production factor for which similar task-level data are available.

3 Model

I modify the Acemoglu and Restrepo (2018)'s task-based approach, where a unique final good is produced combining capital and labor services in a cobb-douglas manner, where the labor services is produced using a continuum of measure one of tasks, $t_{j,t}$ indexed by $j \in [N-1, N]$. On the household side, they supply "green" and "traditional" production factor, in my case two types of skills–green and traditional. These skills are identified in Vona et al. (2018a). I establish a pattern of comparative advantage where tasks are ranked by their greenness, with green-skilled workers being more productive than traditional workers in greener tasks. This classification is conceptually consistent, as Vona et al. (2018a) identify "green skills" precisely based on the capabilities required to perform environmentally sustainable tasks.

The model solution focuses on the equilibrium allocation of skills across tasks, determined by an endogenous threshold J_t. Tasks below this threshold are performed by traditional workers, while those above are performed by green-skilled workers. This allocation has real-world parallels: the lowest range of tasks corresponds to explicitly carbon-intensive occupations (e.g., coal miners, oil rig workers); the intermediate range represents environmentally neutral jobs (e.g., office clerks, retail salespersons); and the highest range encompasses occupations central to economic decarbonization (e.g., solar power engineers, carbon auditors, sustainability officers).

3.1 Discussion of Modeling Choices and Empirical Evidence

This paper adapts an existing model, aka Acemoglu and Restrepo (2018)'s task-based model, studying the structural change of automation to study another key structural transformation on its way–green transition. In this framework, labor is differentiated between "green" and "traditional" based on workers' possession of skills necessary to perform tasks in an environmentally sustainable manner. Importantly, our approach decomposes a sector into its constituent tasks and ranks these tasks by a continuous greenness index. This method captures heterogeneity that is lost in a binary green-dirty classification.

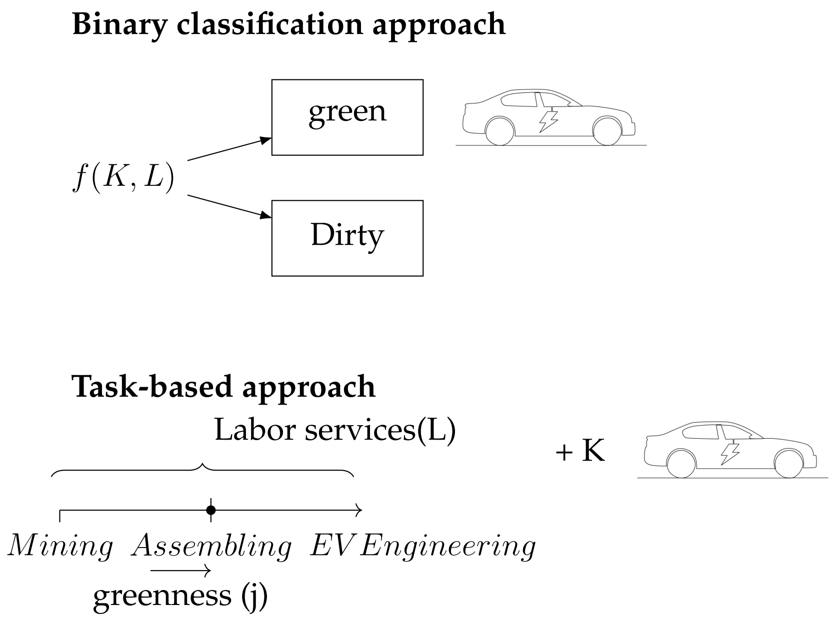

Figure 1. Comparison of Binary Classification vs. Task-based Approaches

3.1.1 Why task-based approach?

Theoretical Basis: Canonical production functions assume fixed roles for production factors. However, the green transition involves fundamental shifts in production methods and required skills. A task-based approach offers granularity by separating production tasks from factors, enabling dynamic reallocation in response to technological and environmental changes. Moreover, by introducing comparative advantage at the task level—where green-skilled labor is relatively more productive in tasks with higher greenness scores—we better capture the nuances of the green transition. This is not merely a reclassification into new sectors; rather, it provides a deeper, continuous measure of environmental performance at the task level.

Data Alignment: According to Vona (2021), the task-based approach provides accurate estimates of green employment. In contrast, binary definitions of green jobs often yield less reliable results.

3.1.2 Greenness Index

The greenness index, calculated by Vona et al. (2018a), quantifies occupations' environmental focus using task data from O*NET. O*NET categorizes tasks into general and specific categories, with the latter further classified into green and traditional tasks through the Green Task Development Project. For example, Metal Sheet Workers perform both green tasks (crafting components for wind turbines) and traditional tasks (operating computerized metalworking equipment). The greenness of an occupation k is measured as:

$$\text{Greenness}_k = \frac{\#\text{green specific tasks}_k}{\#\text{total specific tasks}_k}.$$

This metric reflects an occupation's contribution to environmental sustainability (Vona and Consoli, 2015). Occupations like Chief Sustainability Officer and Solar Power Installers rank high in greenness due to their specialized green tasks. Those with a mix of green and traditional tasks, like Electrical Engineers and Roofers, fall in the middle. Occupations primarily engaged in traditional tasks with sporadic environmental tasks, such as Construction Workers, score lower on the greenness scale.

3.1.3 Green Skills

Following Vona and Consoli (2015), the greenness indicator forms the foundation for a Green General Skills index (GGS), which identifies skills more prevalent in green occupations. In my framework, the labor force is split into "green-skilled labor," which possesses these skills, and "traditional labor," which does not.

3.1.4 Green Skills Environmental Benefits

I assume positive environmental externalities from deploying green inputs rather than traditional inputs across all tasks. This assumption is justified either by positing that green-skilled labor implicitly employs greener technologies, as commonly assumed in the literature (Aghion, Barrage, Hémous, and Liu, 2024), or through the positive externalities of green-skills training documented in management research Usman, Rofcanin, Ali, Ogbonnaya, and Babalola (2023). These positive externalities may include spillover benefits such as improved public health, increased innovation, and enhanced long-term environmental quality.

3.2 Set-up

Now, I have justified my modeling choices, I set-up and solve my theoretical model.

3.2.1 Households

I use a representative household model to capture the demand side of the economy, supplying both green and traditional inputs. Alongside a log-linear utility over consumption and hours worked, I assume there is positive environmental externality of using greener technology, in my case green-skilled labor, over a traditional one. The representative household's problem becomes:

$$\max_{C_t, K_{t+1}, N_t^n, N_t^g} \sum_{t=0}^{\infty} \beta^t u(C_t, N_t^n, N_t^g) = \sum_{t=0}^{\infty} \beta^t (\ln C_t + \eta \ln(1 - N_t^g - N_t^n) + ln\bar{E})$$

$$\text{s.t. } C_t + K_{t+1} = W_t^n N_t^n + W_t^g N_t^g + [R_t - (1+\delta)]K_t \quad \forall t.$$

where C_t is consumption, N_t^g and N_t^n are aggregated working hours of green and traditional workers respectively. Additionally, $\bar{E}$ is a positive externality term from using green input which household takes as given, but it evolves as specified by the following form:

$$E_t = e^{\psi \int_{J_t}^{N} l_{j,t}^g dj}, \quad \psi > 0.$$

Here, $\psi > 0$ captures the additional environmental benefits that arise from using green inputs rather than traditional ones in task production. In other words, every unit of green labor used to perform a task generates a positive spillover—such as reduced pollution or improved public health—that is not reflected in market prices. For simplicity, we assume that this externality enters the household's utility function additively, so that the benefit from using green labor is directly proportional to the amount employed. Since firms do not internalize these external benefits when setting market prices, the competitive equilibrium fails to account for the full social value of green labor, leading to an inefficient allocation of tasks. The household's first-order conditions (FOCs) are derived from this private optimization problem and thus do not incorporate the positive externality, which only affects overall efficiency. The household's FOCs become:

$$W_t^n = \frac{\eta C_t}{1 - N_t^n - N_t^g}, \quad W_t^g = \frac{\eta C_t}{1 - N_t^n - N_t^g}, \quad \beta[R_{t+1} - (1+\delta)] = \frac{C_{t+1}}{C_t}, \quad \forall t.$$

3.2.2 Firms

The final good Y_t is produced by competitive firms combining capital and labor services in a cobb-douglas manner where labor services is produced based on the Acemoglu and Restrepo (2018)'s task-based approach combining a continuum of measure one of tasks, $t_{j,t}$ indexed by $j \in [N-1, N]$.

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}$$

Solving for the profit maximization problem simply gives:

$$R_t = \alpha \cdot \frac{Y_t}{K_t} \quad \text{and} \quad W_t = (1-\alpha) \cdot \frac{Y_t}{L_t}. \tag{1}$$

3.2.3 Labor Services Producer

The labor services L_t is produced based on Acemoglu and Restrepo (2018)'s task-based approach using a continuum of measure one of intermediate inputs, or tasks, $t_{j,t}$ indexed by $j \in [N-1, N]$:

$$L_t = \left(\int_{N-1}^{N} t_{j,t}^{\frac{\chi-1}{\chi}} \, dj \right)^{\frac{\chi}{\chi-1}}. \tag{2}$$

The tasks span from $N-1$ to N, resulting in a constant total number of tasks. The parameter $\chi > 0$ signifies the elasticity of substitution between tasks.

The profit maximization problem of the labor services provider is:

$$\max_{t_{j,t}} W_t L_t - \int_{N-1}^{N} p_{j,t} t_{j,t} \, dj \quad \text{subject to equation (2).}$$

Under perfect competition, the demand function for task j is:

$$t_{j,t} = \left(\frac{p_{j,t}}{W_t} \right)^{-\chi} L_t.$$

The price of aggregated wage W_t is given by:

$$W_t = \left(\int_{N-1}^{N} p_{j,t}^{1-\chi} \, dj \right)^{\frac{1}{1-\chi}}.$$

Based on the derivation in A.3, the price of aggregated task is given by:

$$W_t \quad = \left[\int_{N-1}^{J_t} \left(\frac{\gamma_j^n}{W_{n,t}} \right)^{\chi-1} \, dj + \int_{J_t}^{N} \left(\frac{\gamma_j^g}{W_{g,t}} \right)^{\chi-1} \, dj \right]^{\frac{1}{1-\chi}}. \tag{3}$$

3.2.4 Tasks producers

I arrange tasks between $[N-1, N]$ based on greenness index as computed in Vona, Marin, Consoli, and Popp (2018b) for each occupation (a bundle of tasks, the closest measurable measure of task). Each tasks j can be produced using either green input $l_{j,t}^g$ or traditional input $l^n_{j,t}$, but the two differ both in their relative productivity across tasks and environmental externality. The production function of a generic intermediate input j is:

$$t_{j,t} = \gamma^n_j l^n_{j,t} + \gamma^g_j l^g_{j,t}.$$

It is crucial to note that this production function allows each task to be performed by either green or traditional input, however, the comparative advantage of each type differs across tasks. For each type of input, $i \in \{g, n\}$, γ^i_j represents their task-specific productivity/skills for each task j. These variations in comparative advantage play a significant role in the model. Based on the production structure, the unit costs of producing task j with traditional and green inputs respectively are:

$$p^n_{j,t} = \frac{W^n_t}{\gamma^n_j} \quad \text{and} \quad p^g_{j,t} = \frac{W^g_t}{\gamma^g_j}.$$

3.2.5 Assumption 1: $\frac{\gamma^g_j}{\gamma^n_j}$ is continuously differentiable and strictly increasing in j

Tasks are ranked by their greenness index, and each can be done by either green or traditional labor. As explained previously, since green skills are defined by this index, green-skilled labor has a growing advantage over traditional labor for tasks with a higher greenness index, meaning $\frac{\gamma^g_j}{\gamma^n_j}$ is increasing in j.

Lemma 1. *In any equilibrium there exist $J_t \in (N-1, N)$ such that for any $j < J_t, l^g_{j,t} = 0$ and for any $j > J_t, l^n_{j,t} = 0$.*[1]

Because the allocation of tasks to factors is cost-minimizing and because $\frac{\gamma^g_j}{\gamma^n_j}$ is (strictly) increasing, there exists a threshold J_t such that all tasks below the threshold are produced with tradition labor and those above it will be produced with green-skilled labor. It is determined in the model as:

$$\frac{W^n_t}{\gamma^n_{J_t}} = \frac{W^g_t}{\gamma^g_{J_t}} \implies J_t : \frac{W^n_t}{W^g_t} = \frac{\gamma^g_{J_t}}{\gamma^n_{J_t}}. \tag{4}$$

Based on the derivation in A.3, and the demand for task in the above section, the factor

[1] The proof is similar to Acemoglu and Zilibotti (2001).

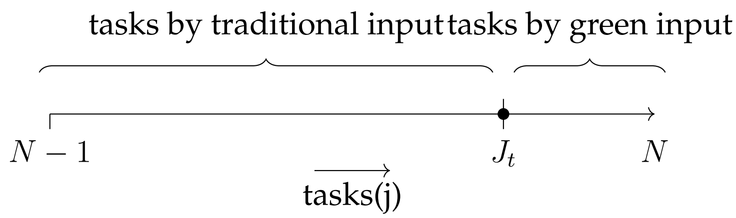

Figure 2. The task space indexed based on greenness(j)

demand are given by:

$$l^n_{j,t} = \begin{cases} (\gamma^n_j)^{\chi-1} W^{n-\chi}_t L_t & \text{if } j \in [N-1, J_t] \\ 0 & \text{if } j \in (J_t, N] \end{cases} \tag{5}$$

$$l^g_{j,t} = \begin{cases} 0 & \text{if } j \in [N-1, J_t] \\ (\gamma^g_j)^{\chi-1} W^{g-\chi}_t L_t & \text{if } j \in (J_t, N] \end{cases} \tag{6}$$

Thus, the production function of the task producer j takes the following form:

$$t_{j,t} = \left\{ \begin{array}{cc} \gamma^n_j l^n_{j,t}, & \text{if } j \in [N-1, J_t] \\ \gamma^g_j l^g_{j,t}, & \text{if } j \in (J_t, N] \end{array} \right\}. \tag{7}$$

3.2.6 Aggregation

Here L^n_t and L^g_t is derived by aggregating the demand for green and traditional labor from this expression, I have the following aggregate demand for the two types of labor[2]:

$$L^n_t = L_t W^{n-\chi}_t \int_{N-1}^{J_t} (\gamma^n_j)^{\chi-1}\, dj \tag{8}$$

$$\implies W^n_t = \left(\frac{L_t}{L^n_t}\right)^{\frac{1}{\chi}} \left(\int_{N-1}^{J_t} (\gamma^n_j)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} = L_{L^n} \tag{9}$$

$$L^g_t = L_t W^{g-\chi}_t \int_{J_t}^{N} (\gamma^g_j)^{\chi-1}\, dj \tag{10}$$

$$\implies W^g_t = \left(\frac{L_t}{L^g_t}\right)^{\frac{1}{\chi}} \left(\int_{J_t}^{N} (\gamma^g_j)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} = L_{L^g}. \tag{11}$$

[2]Look Appendix A.4 for the derivation.

Here, the ratio of aggregate demand for green to traditional labor is given by:

$$\frac{L_t^g}{L_t^n} = \frac{\int_{J_t}^{N} (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^{J_t} (\gamma_j^n)^{\chi-1}\, dj} \left(\frac{W_t^g}{W_t^n}\right)^{-\chi}.$$

The resulting relative factor productivity for labor is:

$$\frac{W_t^g}{W_t^n} = \left(\frac{L_t^n}{L_t^g}\right)^{\frac{1}{\chi}} \left(\frac{\int_{J_t}^{N} (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^{J_t} (\gamma_j^n)^{\chi-1}\, dj}\right)^{\frac{1}{\chi}}. \tag{12}$$

3.2.7 Optimal aggregated output

Aggregating across tasks gives the following formula for the aggregate labor services:

$$L_t = \left(\left(\int_{N-1}^{J_t} (\gamma_j^n)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} (L_t^n)^{\frac{\chi-1}{\chi}} + \left(\int_{J_t}^{N} (\gamma_j^g)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} (L_t^g)^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}}.[3] \tag{13}$$

The equation resembles a CES production function, where the combining two types of labor, with elasticity of substitution χ, generates aggregate output. The crucial insight in the task-based production is that the share parameter for each labor is endogenous that depends on several factors: the task distribution threshold (J_t), task-specific productivity schedules (γ_n and γ_g), and the elasticity of substitution χ. Increasing J_t directs more tasks to traditional labor relative to green-skilled labor in equilibrium.

3.2.8 Competitive Market Task Allocation

Based on the cost-minimizing task cut-off condition in equation (4) and the relative wage expression in equation (12), we derive the following expression for the endogenously determined task allocation threshold in competitive equilibrium:

$$J_t : \frac{\gamma_{J_t}^g}{\gamma_{J_t}^n} = \frac{W_t^g}{W_t^n} = \left(\frac{L_t^n}{L_t^g}\right)^{\frac{1}{\chi}} \left(\frac{\int_{J_t}^{N} (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^{J_t} (\gamma_j^n)^{\chi-1}\, dj}\right)^{\frac{1}{\chi}}. \tag{14}$$

[3] See appendix A.4 for the derivation.

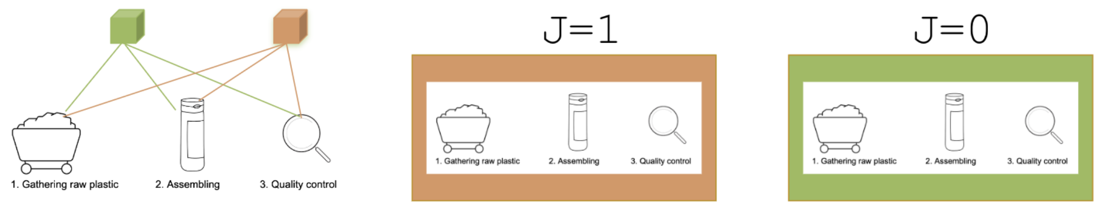

Figure 3. Understanding the endogenous threshold J

The figure illustrates how the threshold J governs task allocation: although any task can be performed by either green or traditional inputs, a value of $J = 1$ implies all tasks are performed by traditional inputs, while $J = 0$ implies all are done by green inputs. In equilibrium, J lies strictly between 0 and 1, reflecting a cost-driven allocation of tasks between the two input types.

Here, the left hand side of eq(14) is continuous and increasing in J_t by assumption 3.2.5 and the right hand side is decreasing in J_t, which implies the above equation gives a unique solution J_{ce} defined by the implicit function:

$$F = \frac{\gamma^g_{J_t}}{\gamma^n_{J_t}} - \left(\frac{L^n_t}{L^g_t}\right)^{\frac{1}{\chi}} \left(\frac{\int_{J_t}^{N} (\gamma^g_j)^{\chi-1}\, dj}{\int_{N-1}^{J_t} (\gamma^n_j)^{\chi-1}\, dj}\right)^{\frac{1}{\chi}} \text{ s.t. } J_t = J_t(L^n_t, L^g_t, \gamma^g_j, \gamma^n_j, \chi).$$

Using implicit function theorem I get:

$$\frac{dJ_t}{dL^n_t} = -\frac{\frac{\partial F}{\partial L^n_t}}{\frac{\partial F}{\partial J_t}}, \frac{dJ}{dL^g_t} = -\frac{\frac{\partial F}{\partial L^g_t}}{\frac{\partial F}{\partial J_t}}.$$

The key takeaway here is that how the competitive task allocation threshold J_t moves depends on $\frac{\partial F}{\partial J_t}$ which depends on the shape of the relative productivity schedules $\frac{\gamma_{J^g_t}}{\gamma_{J^n_t}}$ at the task allocation threshold J_t, which I will elaborate more throughout the paper.

3.3 Equilibrium

For any range of tasks $[N-1, N]$, I consider the steady state equilibrium characterized by 11 equilibrium variables $\{C, Y, K, L, L^n, L^g, R, W, W^n, W^g, J\}$ and the following 11 equilibrium equations:

1. $Y = K^\alpha L^{1-\alpha}$
2. $R = \alpha \frac{Y}{K}$
3. $W = (1-\alpha)\frac{Y}{L}$

4. $W = \left[\int_{N-1}^{J} \left(\frac{W^n}{\gamma_j^n}\right)^{1-\chi} dj + \int_{J}^{N} \left(\frac{W^g}{\gamma_j^g}\right)^{1-\chi} dj\right]^{\frac{1}{1-\chi}}$

5. $L = \left[\left(\int_{N-1}^{J} (\gamma_j^n)^{\chi-1} dj\right)^{\frac{1}{\chi}} (L^n)^{\frac{\chi-1}{\chi}} + \left(\int_{J}^{N} (\gamma_j^g)^{\chi-1} dj\right)^{\frac{1}{\chi}} (L^g)^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}$

6. $W^n = \left(\frac{L}{L^n}\right)^{\frac{1}{\chi}} \left(\int_{N-1}^{J} (\gamma_j^n)^{\chi-1} dj\right)^{\frac{1}{\chi}}$

7. $W^g = \left(\frac{L}{L^g}\right)^{\frac{1}{\chi}} \left(\int_{J}^{N} (\gamma_j^g)^{\chi-1} dj\right)^{\frac{1}{\chi}}$

8. $J : \frac{W^g}{W^n} = \frac{\gamma_J^g}{\gamma_J^n}$

9. $R = \frac{1}{\beta} - 1 + \delta$

10. $W^g = \frac{C}{\eta(1-L^g-L^n)}$

11. $W^n = \frac{C}{\eta(1-L^g-L^n)}$

12. $Y = C$

The key variable of interest in the task-based model is the endogenous task allocation threshold J which is defined uniquely in the equilibrium.

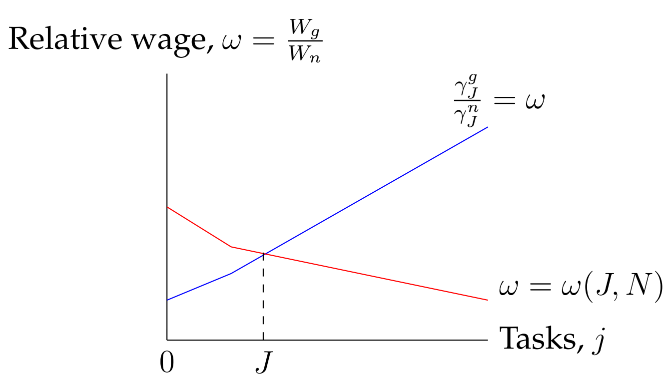

Figure 4. Static Equilibrium

In the above equilibrium diagram, the upward-sloping curve represents the relative productivity schedules of green to traditional labor that is increasing in j by assumption and the downward sloping curve is derived from the relative demand for green-skilled labor given in equation (11).

3.4 Social Planner's Problem

I now consider the social planner's problem who can freely choose the allocation of different types of labor to find the first best solution. The social planner internalizes the environment externalities green input generates and chooses the demand of traditional and green input for each task deciding the final allocation of the two types of input. The first best solution is given by solving the following maximization problem:

$$\max_{\{l^n_{j,t}, l^g_{j,t}, J_t \in [N-1,N]\}} \ln Y_t + \ln\left(1 - \int_{N-1}^{J_t} l^n_{j,t} dj - \int_{J_t}^{N} l^g_{j,t} dj\right) + \psi \int_{J_t}^{N} l^g_{j,t} dj$$

$$\text{s.t. } Y_t = \left(\int_{N-1}^{J_t} (\gamma^n_j l^n_{j,t})^{\frac{\chi-1}{\chi}} \, dj + \int_{J_t}^{N} (\gamma^g_j l^g_{j,t})^{\frac{\chi-1}{\chi}} \, dj\right)^{\frac{\chi}{\chi-1}}, \quad l^n_{j,t} \geq 0, \quad l^g_{j,t} \geq 0.$$

Taking first order conditions and solving for J_{spp}, I show in the appendix B.2 that $J_{sp} < J_{ce}$.

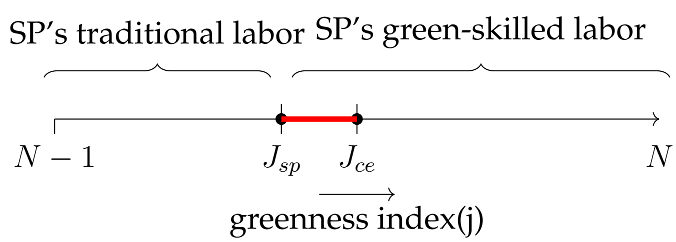

Figure 5. The task space showing $J_{sp} < J_{ce}$.

Proposition 1. *In addition to effective cost in terms of cost and productivity, when a social planner also takes into account the environmental externality, optimal allocation implies allocation of more tasks to green inputs relative to traditional inputs.*[4]

3.5 Implementation

In this section, I design task-specific green wage subsidies to implement a lower task-allocation threshold J_{sp}. The subsidy is intended to offset the productivity disadvantage of green inputs in certain tasks by incorporating their environmental benefits. This design explicitly weighs the trade-off between potential productivity losses (when green inputs are less efficient) and significant environmental gains. In cases with large productivity differences but also large environmental benefits, the subsidy must balance these factors to ensure net welfare gains.

[4] See appendix B for the derivation.

Proposition 2. *A social planner can implement a lower task-allocation threshold $J_{sp} < J_{ce}(L^n, L^g)$ using the following task-specific green input subsidy:*[5]

$$\tau_j^g = \begin{cases} 0 & \textit{if } j \leq J_{sp} \\ 1 - \frac{Y_{L^n}}{Y_{L^g}} \frac{\gamma^n(j)}{\gamma^g(j)+\psi} \geq 0 & \textit{if } j > J_{sp} \end{cases}$$

The main logic behind this policy design is that the subsidy increases the cost-competitiveness of green input for tasks where it's less productive than traditional input, considering the added environmental benefits, thereby promoting its usage and helping achieve the social planner's goal of a lower J_{spp}.

3.5.1 Comparative statics of subsidy

Normalizing the traditional productivity $\gamma^n(j)$ equal to 1 and writing the size of the subsidy with respect of input cost premium $\bar{W} = \frac{W^g}{W^n}$, relative productivity schedule $\bar{\gamma} = \frac{\gamma_j^g}{\gamma_j^n}$, and environmental externality ψ, we get:

- With respect to relative input cost premium, $\bar{W} : \frac{\partial \tau_j^g}{\bar{W}} = \frac{1}{\bar{W}^2(\bar{\gamma}+\psi)}$ i.e. the subsidy size increases with the cost premium between the green input and the traditional input it's trying to substitute for is higher.
- With respect to relative productivity, $\bar{\gamma} : \frac{\partial \tau_j^g}{\bar{\gamma}} = \frac{1}{\bar{W}(\bar{\gamma}+\psi)^2}$ i.e. the subsidy size increases with the relative productivity gap between the green and the traditional input.
- With respect to environmental externality, $\psi : \frac{\partial \tau_j^g}{\psi} = \frac{1}{\bar{W}(\bar{\gamma}+\psi)^2}$ i.e. the subsidy size increases with the size of the externality.

3.6 Effectiveness of target green input subsidies

The main takeaway so far is that the competitive market decides whether to deploy traditional or green inputs across tasks purely based on their effective cost and productivities, without considering the environmental externality of using green input over the traditional one. So, if the green transition necessitates allocating more tasks of production process to green inputs, akin to climate policy tools used in the U.S., such as those in the Inflation Reduction Act (IRA), I consider the targeted subsidy to the green input cost and assess its effectiveness in promoting the use of green inputs under this framework.

[5] See appendix B.3 for the details about the implementability of τ_j^g.

3.6.1 Green input subsidy, τ^g

In a canonical model, either providing cost subsidy τ^g for green input or imposing tax τ^n for traditional input would reduce the effective relative cost of deploying green input for firms, which would in turn increase the demand and equilibrium use of green inputs. In case of the task-based approach, because of the task allocation effect, the exact effect of subsidy is not straightforward.

Proposition 3. *The effectiveness of green wage subsidy in task allocation depends on* $\epsilon_{\frac{\gamma_J^g}{\gamma_J^n},J}$*, the elasticity of the comparative advantage schedule at the task threshold* J*.*

Adding green wage subsidy τ^g to the endogenous task threshold equation in (3):

$$J : \frac{\gamma_J^g}{\gamma_J^n} = \frac{(1-\tau^g)W^g}{W^n} = (1-\tau^g), \text{where at } J_{ce} \text{ by HH's FOC.}$$

This means J is defined implicity by: $F = \frac{\gamma_J^g}{\gamma_J^n} - (1-\tau^g)$.
By implicit function theorem:

$$\frac{dJ}{d\tau^g} = -\frac{\frac{\partial F}{\partial \tau^g}}{\frac{\partial F}{\partial J}} = \frac{-1}{\epsilon_{\frac{\gamma_J^g}{\gamma_J^n},J}}.$$

Here, what is new in the task-based approach is that the effectiveness of the wage subsidy on moving the task-allocation threshold depends on $\epsilon_{\frac{\gamma_J^g}{\gamma_J^n},J}$ which depends on the functional form of the comparative advantage schedule.

4 Calibration and Simulation

4.1 Calibration

The qualitative analysis demonstrates that the functional form of comparative advantage schedules between green and traditional inputs critically affects the effectiveness of targeted subsidies in task allocation. To quantify this mechanism, I conduct a calibration exercise using empirical data. The model is calibrated to an initial steady state without subsidies. Standard economic parameters follow established conventions in the real business cycle literature: capital share (α) is set to 0.33, discount rate (β) to 0.99, and capital depreciation rate (δ) to 0.025. The labor disutility weight (η) is normalized to 1, reflecting the conventional balance between labor supply and leisure.

The model introduces several novel parameters. The elasticity of substitution between green and traditional units (χ) is set to 1.5, based on Papageorgiou, Saam, and Schulte (2017), who estimate this parameter to be greater than 1 and close to 2, consistent with Acemoglu and Restrepo (2019). The positive externality parameter (ψ), representing the weight of environmental quality relative to private consumption, is set to 0.4 following Angelopoulos et al. (2010), justified as being towards the upper bound typically assigned to public goods in comparable utility functions. This corresponds to a 1.4% increase in consumption equivalent. I conduct sensitivity analyses with varying values of this parameter.

The comparative advantage schedules for green and traditional inputs, γ_j^g and γ_j^n, are fundamental to the production structure. I employ the following general formulation for the relative productivity schedule:

$$\frac{\gamma_J^g}{\gamma_J^n} = \frac{A \cdot J^{\nu_g}}{B \cdot (1-J)^{\nu_n}} \text{ with } \nu_g \geq 0, \nu_n \geq 0, \nu_g + \nu_n > 0.$$

Following Acemoglu and Restrepo (2018) and Acemoglu, Manera, and Restrepo (2020a), I normalize the productivity of the traditional input to 1 and consider the case $\frac{\gamma_J^g}{\gamma_J^n} = A \cdot J^{\nu}$.

I calibrate the model incorporating two key empirical findings from Vona et al. (2018a): green occupations are typically higher-skill and less routine-intensive than non-green occupations. Based on these insights, I calibrate ν, the parameter governing comparative advantage of green labor across tasks, using two reference points: routine-intensity distribution from automation literature and skill-intensity distribution from skill-biased technical change literature. Following Acemoglu et al. (2020a), I use $\nu = 2.12$ for routine intensity, and following Marczak, Beissinger, and Brall (2022), I use $\nu = 0.67$ for skill intensity. These values allow me to examine both concave and convex productivity schedules, which significantly impact policy effectiveness.

For both specifications, I calibrate parameter A in the function $\frac{\gamma_J^g}{\gamma_J^n}$ to match two U.S. empirical targets: (i) task allocation threshold J (the share of tasks performed by traditional versus green-skilled labor) set at 0.806, based on Bowen, Kuralbayeva, and Tipoe (2018)'s estimate that green jobs constitute 19.4% of employment, and (ii) the green wage premium $\frac{W_g}{W_n}$ of 2%, derived from Shibata, Mano, and Bergant (2022) estimates.

After calibrating the model, I simulate the impact of green input subsidies of varying sizes on reducing the task allocation threshold favoring green inputs for two calibrated productivity schedules. The results are presented below.

4.2 Simulation and Policy Analysis

Policy Design: How to design subsidies to improve effectiveness?

- **Exercise:** Fix the total subsidy budget, vary task-specific green wage subsidies under concave vs. convex relative-productivity schedules (ν), and record the resulting task threshold J and green-input share.
- **Goal:** Identify where each subsidy dollar shifts the most tasks to green inputs.
- **Result:** Subsidies are most effective when the productivity gap is narrowest.

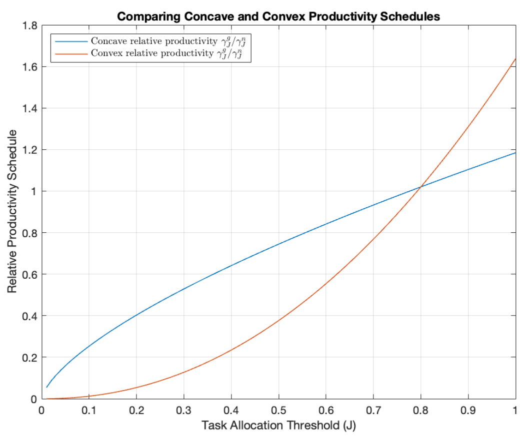

Figure 6. Relative Productivity

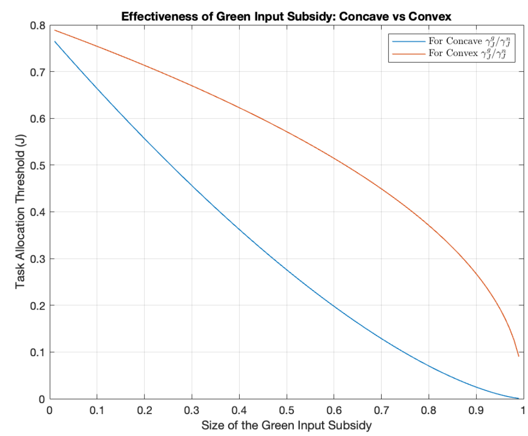

Figure 7. Effective Methods

The simulations show how subsidy effectiveness depends on relative productivity and the initial task cutoff J:

- *Concave schedule* $\left(\frac{\gamma_J^g}{\gamma_J^n}\mathit{concave}\right)$*:* At low J, a small drop in J causes a large productivity loss for green inputs, so green demand rises only modestly when costs fall. At higher J, the same cost drop yields a much larger increase in green-input demand.
- *Convex schedule* $\left(\frac{\gamma_J^g}{\gamma_J^n}\mathit{convex}\right)$*:* At low J, you need a big reduction in J to lower relative productivity, so subsidies have little effect until J falls enough. At higher J, cost changes more directly boost green demand.

Figure 8 highlights this relationship. Starting from a high steady-state $J = 0.8$, the productivity gap is narrower under the concave shape than under the convex shape, making a 20% subsidy (comparable to Casey et al. (2023)) more effective for the concave case.

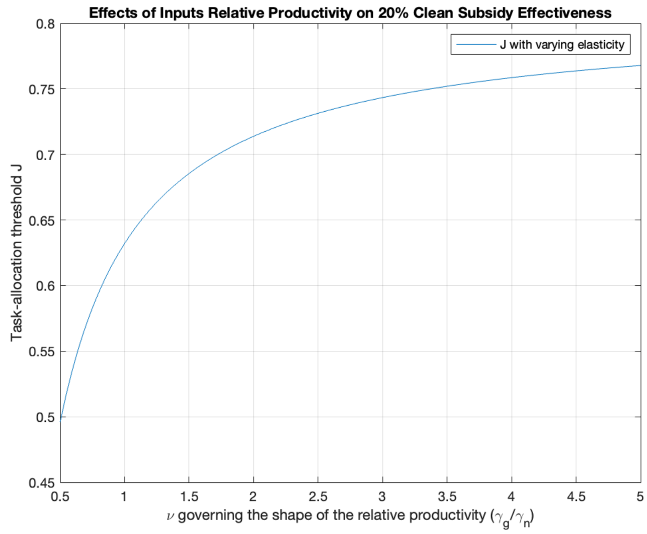

Figure 8. Varying effectiveness of relative productivity parameter ν on task threshold J

4.3 Welfare Analysis

A central question in climate-macro research concerns optimal subsidy financing methods and their general equilibrium welfare implications. This section presents a comprehensive welfare analysis of green input subsidies under different financing schemes. I employ numerical solutions to track changes in model variables between steady states and quantify their impact on overall welfare as percentage changes relative to the initial (no-subsidy) steady state.

I calculate the necessary percentage change in initial consumption (ω) to equalize welfare across states:

$$W_l = (1 - \omega) * 100$$

$$\text{s.t.,} \quad \{[ln(\omega C) + \eta \ln(1 - L^g - L^n) + \psi L^g] - [ln(C') + \eta \ln(1 - L^{g'} - L^{n'}) + \psi L^{g'}]\} = 0.$$

The value of W_l provides a welfare comparison metric: a positive W_l indicates higher aggregate welfare in the initial steady state, meaning consumption would need to be reduced by W_l percent to match the utility level in the new steady state.

4.3.1 Productivity vs. Environmental Tradeoff

Reducing the task allocation threshold J involves a fundamental tradeoff: positive environmental benefits come at the cost of productivity losses as tasks are reallocated to relatively less productive factors. In scenarios where the productivity gap is large but the environmental benefit (captured by ψ) is also significant, the net welfare impact depends on the balance between these effects. Consequently, welfare gains depend critically on the magnitude of positive environmental externalities.

Policy Trade-off: How large must the environmental externality be for subsidies to improve welfare?

- **Exercise:** Fix a 5 % green-skilled labor subsidy and vary the externality parameter ψ. Compute total welfare changes using

$$\frac{\partial \mathcal{W}}{\partial J} = \underbrace{\frac{\partial Y}{\partial J}}_{\text{productivity effect}} - \underbrace{\frac{\eta}{1 - L^n - L^g}\left[\frac{\partial L^g}{\partial J} + \frac{\partial L^n}{\partial J}\right]}_{\text{labor reallocation effect}} + \underbrace{\psi \cdot \frac{\partial L^g}{\partial J}}_{\text{environmental benefit}}.$$

  Plot welfare versus ψ (Figure 9) and calculate the consumption-equivalent gain at the cutoff.

- **Goal:** Identify the minimum ψ (environmental-externality magnitude) at which the 5 % subsidy yields a positive welfare change.

- **Result:** Welfare becomes positive only when $\psi \approx 1.2$, equivalent to a 4.3 % consumption-equivalent gain.

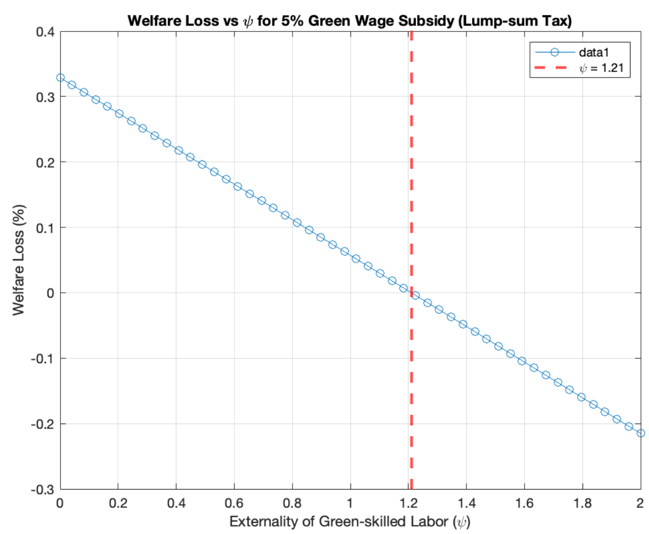

Figure 9. Welfare for different values of externality parameter ψ

From figure 9, we see that the welfare change becomes negative (i.e., a consumption-equivalent gain) once ψ reaches about 1.2. Below $\psi = 1.2$, welfare losses are positive—meaning the subsidy's productivity costs exceed its environmental benefits—whereas at and above $\psi = 1.2$, the welfare loss turns negative, indicating a net welfare gain. This threshold is more than twice the range (0.2–0.5) typically used for public-good parameters in the macro-public literature. To illustrate this cutoff intuitively, I compute its consumption equivalent: at $\psi = 1.2$, the positive externality of green-skilled labor is equivalent to a 4.3% increase in consumption.

The elasticity of substitution χ between tasks strongly affects this result: higher χ makes it easier to shift away from polluting tasks, so a given externality yields larger welfare gains. The key role of χ in climate-macro analysis is well documented (e.g., Acemoglu, Aghion, Bursztyn, and Hemous, 2012; Casey et al., 2023; Cruz and Rossi-Hansberg, 2024). To illustrate, I vary χ from 0.5 to 2 and calculate the consumption-equivalent value at $\psi = 1.21$ for each case.

Figure 10 shows that when χ is low (around 0.5–1.0), tasks are hard to reallocate, so the same externality yields a small or even negative welfare gain. As χ rises above 1, tasks become easier to switch from "dirty" to "green," and the welfare boost grows. In other words, for a fixed $\psi = 1.21$, the consumption-equivalent benefit increases with χ, confirming that greater task substitutability amplifies the welfare payoff of the externality.

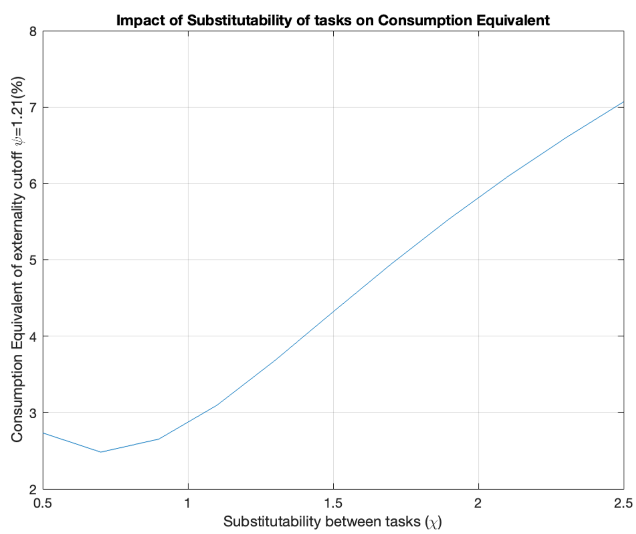

Figure 10. Consumption equivalent of $\psi = 1.21$ vs. elasticity of substitution of tasks

Policy Financing: Which tax base funds subsidies with the least welfare loss?

- **Exercise:** For subsidies of 1–10 % of output, compute the required tax rates and welfare costs under three financing methods—labor-income tax, capital-income tax, and lump-sum tax. Present tax rates in Figure 11 and welfare losses in Figure 12.

- **Goal:** Determine which tax base generates the smallest distortion for funding green subsidies.

- **Result:** Lump-sum taxation yields the lowest welfare loss, capital taxation is second-best, and labor taxation is most costly.

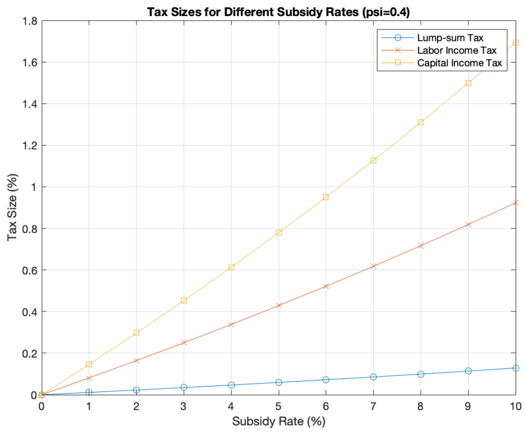

Figure 11. Tax Sizes for Different Subsidy Rates (psi=0.4)

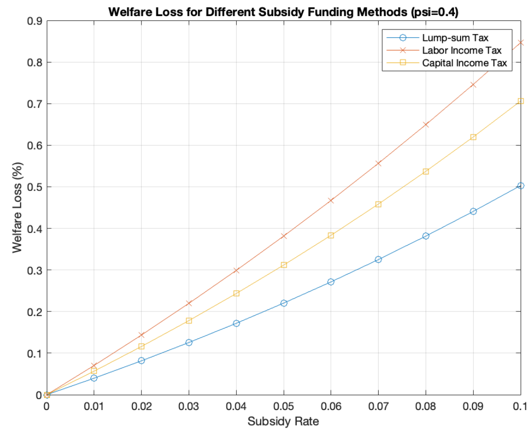

Figure 12. Welfare costs for different financing methods

The first figure shows that the capital-tax rate must be set highest to fund a given subsidy, followed by the labor-tax rate, while the lump-sum charge is lowest. In the second figure, interestingly financing green subsidies with a labor-income tax produces larger welfare losses than using a capital-income tax, which counteracts the classic Chamley–Judd result that capital should bear no tax in the long run because it is the most distortionary base (Chamley, 1986; Judd, 1985). In this task-based model, labor is the factor most closely tied to adopting green inputs; taxing it directly raises the marginal cost of completing greener tasks and amplifies the misallocation. A capital-income tax, by contrast, is partly absorbed by traditional inputs and therefore distorts the greenness margin less.

5 Conclusion

This paper presents a task-based general-equilibrium model with a continuous greenness index to evaluate targeted green-input subsidies. By modeling production as a continuum of tasks rather than a binary "green versus dirty" split, the framework uncovers a market failure: competitive markets allocate too few tasks to green inputs, justifying corrective subsidies. The model is structured to answer three key aspects of subsidy (fiscal) policy—design, trade-off, and financing—by showing where subsidies are most effective, under what environmental-productivity conditions they improve welfare, and which tax base funds them with the least distortion.

First, subsidy effectiveness hinges on the productivity gap between green and traditional inputs. When that gap is narrow, even modest subsidies can shift many tasks toward greener methods. Policymakers should therefore target sectors where green inputs already approach traditional productivity and pair subsidies with R&D or infrastructure support to close any remaining gaps. *Second,* calibrated to U.S. data, a 5% green-labor subsidy yields net welfare gains only if the environmental externality parameter ψ exceeds about 1.2—roughly three times the standard public-goods benchmark—and corresponds to a 4.3% consumption-equivalent benefit. This threshold falls as the substitution elasticity χ rises, since higher χ makes it easier to switch tasks to green inputs. *Third*, among lump-sum, labor-income, and capital-income taxes, lump-sum financing imposes the smallest welfare loss, capital taxes are second-best, and labor taxes are most distortionary.

These findings can help shape fiscal policy for the green transition. By showing where subsidies have the biggest environmental and economic impact, governments can design more efficient green stimulus programs that ease budget pressure and avoid financial strain. Future work could expand the model to include changing productivity, other inputs (like capital or energy), and explore how green subsidies interact with monetary and macroprudential policies in this framework.

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Appendix

Note: All time indices have been removed from the derivations for simplicity.

A Firms

A.1 Final goods

I start by solving for the profit maximization of the final goods producer.

$$\max_{K,L} P(K^{\alpha}L^{1-\alpha}) - RK - WL$$

Normalizing the final price P to 1 and taking FOCs w.r.t. K and L gives:

$$R = \alpha \cdot \frac{Y}{K} \quad \text{and} \quad W = (1-\alpha) \cdot \frac{Y}{L}.$$

A.2 Labor Services

A.2.1 Demand for task j

First, I find a demand function for t_j by minimizing consumption cost.

$$\max_{t_j} WL - \int_{N-1}^{N} p_j t_j \, dj$$

$$\max_{t_j} W \left(\int_{N-1}^{N} t_j^{\frac{\chi-1}{\chi}} \, dj \right)^{\frac{\chi}{\chi-1}} - \int_{N-1}^{N} p_j t_j \, dj$$

Taking FOCs w.r.t. t_j gives:

$$t_j = \left(\frac{p_j}{W} \right)^{-\chi} \quad \text{,where} \quad W = \left(\int_{N-1}^{N} p_j^{1-\chi} \, dj \right)^{\frac{1}{1-\chi}}.$$

A.3 Task aggregator

A.3.1 Demand for different tasks

I will first derive the demand for different factors for each task.

$$\max_{l_j^g, l_j^n} WL - \int_{N-1}^{J} W_n l_j^n \, dj - \int_{J}^{N} W_g l_j^g \, dj$$

$$\text{s.t. } L = \left(\int_{N-1}^{J} (\gamma_j^n l_j^n)^{\frac{\chi-1}{\chi}} \, dj + \int_{J}^{N} (\gamma_j^g l_j^g)^{\frac{\chi-1}{\chi}} \, dj \right)^{\frac{\chi}{\chi-1}}$$

Taking FOCs give:

$$\frac{W_n}{W} = \gamma_j^n \cdot (\gamma_j^n l_j^n)^{\frac{-1}{\chi}} \implies l_j^{n*} = (\gamma_j^n)^{\chi-1} \left(\frac{W_n}{W} \right)^{-\chi} L$$

$$\frac{W_g}{W} = \gamma_j^g \cdot (\gamma_j^g l_j^g)^{\frac{-1}{\chi}} \implies l_j^{g*} = (\gamma_j^g)^{\chi-1} \left(\frac{W_g}{W} \right)^{-\chi} L$$

Therefore, the optimal green input cost is:

$$\begin{aligned} \int_{J}^{N} W_g l_j^{g*} \, dj &= \int_{J}^{N} W_g (\gamma_j^g)^{\chi-1} \left(\frac{W_g}{W} \right)^{-\chi} L \, dj \\ &= \int_{J}^{N} (\gamma_j^g)^{\chi-1} \, dj \; W_g^{1-\chi} \, W^{\chi} L \end{aligned}$$

Finally, the optimal non-green input cost is:

$$\begin{aligned} \int_{N-1}^{J} W_n l_j^{n*} \, dj &= \int_{N-1}^{J} W_n (\gamma_j^n)^{\chi-1} \left(\frac{W_n}{W} \right)^{-\chi} L \, dj \\ &= \int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj \; W_n^{1-\chi} \, W^{\chi} L \end{aligned}$$

A.3.2 Task aggregator price

I now turn to deriving the expression for the price for an aggregated task, W, by solving the cost minimization problem. I assume a perfectly competitive market.

$$
\begin{aligned}
&WL = \int_{N-1}^{J} W_n l_j^n \, dj + \int_{J}^{N} W_g l_j^g \, dj \\
&= \int_{N-1}^{J} W_n (\gamma_j^n)^{\chi-1} \left(\frac{W_n}{W}\right)^{-\chi} L \, dj + \int_{J}^{N} W_g (\gamma_j^g)^{\chi-1} \left(\frac{W_g}{W}\right)^{-\chi} L \, dj \\
&= W_n^{1-\chi} \frac{L}{W^{-\chi}} \int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj + W_g^{1-\chi} \frac{L}{W^{-\chi}} \int_{J}^{N} (\gamma_j^g)^{\chi-1} \, dj \\
&\implies W^{1-\chi} = W_n^{1-\chi} \int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj + W_g^{1-\chi} \int_{J}^{N} (\gamma_j^g)^{\chi-1} \, dj \\
&\therefore W = \left[W_n^{1-\chi} \int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj + W_g^{1-\chi} \int_{J}^{N} (\gamma_j^g)^{\chi-1} \, dj \right]^{\frac{1}{1-\chi}} .
\end{aligned}
$$

A.4 Aggregation

A.4.1 Aggregate demand of labor

I have from the above sections alongside the normalized $l_j^{n*} = (\gamma_j^n)^{\chi-1} \left(\frac{W_n}{W}\right)^{-\chi} L$ and $l_j^{g*} = (\gamma_j^g)^{\chi-1} \left(\frac{W_g}{W}\right)^{-\chi} L$. Aggregating across non-green tasks $[N-1, J]$, I get the aggregate demand for the non-green labor:

$$
\begin{aligned}
L^n &= \int_{N-1}^{J} l_j^{n*} \, dj \\
&= \int_{N-1}^{J} (\gamma_j^g)^{\chi-1} \left(\frac{W_g}{W}\right)^{-\chi} L \, dj \\
\implies L^n &= \left(\frac{W_g}{W}\right)^{-\chi} L \int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj \\
\implies W_n &= \left(\frac{Y}{L^n}\right)^{\frac{1}{\chi}} \left(\int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj \right)^{\frac{1}{\chi}}
\end{aligned}
$$

Aggregating across green tasks $(J, N]$, I get the aggregate demand for the green labor:

$$\begin{aligned}
L^g &= \int_J^N l_j^{g*}\, dj \\
&= \int_J^N (\gamma_j^g)^{\chi-1} W_g^{-\chi} Y\, dj \\
\Longrightarrow L^g &= Y W_g^{-\chi} \int_J^N (\gamma_j^g)^{\chi-1}\, dj \\
\Longrightarrow W_g &= \left(\frac{Y}{L^g}\right)^{\frac{1}{\chi}} \left(\int_{N-1}^J (\gamma_j^g)^{\chi-1}\, dj\right)^{\frac{1}{\chi}}
\end{aligned}$$

The ratio of aggregate demand for green to non-green labor is given by:

$$\begin{aligned}
\frac{L^g}{L^n} &= \frac{Y W_g^{-\chi} \int_J^N (\gamma_j^g)^{\chi-1}\, dj}{Y W_n^{-\chi} \int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj} \\
&= \frac{\int_J^N (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj} \left(\frac{W_g}{W_n}\right)^{-\chi}
\end{aligned}$$

And, the resulting relative factor productivity for labor is:

$$\frac{W_g}{W_n} = \left(\frac{L^n}{L^g}\right)^{\frac{1}{\chi}} \left(\frac{\int_J^N (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj}\right)^{\frac{1}{\chi}}$$

Since the final good is the numeraire, and the final good price P is normalized to 1:

$$\begin{aligned}
P &= 1 = \left[W_n^{1-\chi} \int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj + W_g^{1-\chi} \int_J^N (\gamma_j^g)^{\chi-1}\, dj\right]^{\frac{1}{1-\chi}} \\
\Longrightarrow 1 &= W_n^{1-\chi} \int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj + W_g^{1-\chi} \int_J^N (\gamma_j^g)^{\chi-1}\, dj \\
\Longrightarrow 1 &= \left(\left(\frac{Y}{L^n}\right)^{\frac{1}{\chi}} \left(\int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj\right)^{\frac{1}{\chi}}\right)^{1-\chi} \int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj + \\
&\quad \left(\left(\frac{Y}{L^g}\right)^{\frac{1}{\chi}} \left(\int_J^N (\gamma_j^g)^{\chi-1}\, dj\right)^{\frac{1}{\chi}}\right)^{1-\chi} \int_J^N (\gamma_j^g)^{\chi-1}\, dj \\
\Longrightarrow 1 &= \left(\frac{Y}{L^n}\right)^{\frac{1-\chi}{\chi}} \left(\int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} + \left(\frac{Y}{L^g}\right)^{\frac{1-\chi}{\chi}} \left(\int_J^N (\gamma_j^g)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} \\
\therefore Y &= \left(\left(\int_{N-1}^J (\gamma_j^n)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} (L^n)^{\frac{\chi-1}{\chi}} + \left(\int_J^N (\gamma_j^g)^{\chi-1}\, dj\right)^{\frac{1}{\chi}} (L^g)^{\frac{\chi-1}{\chi}}\right)^{\frac{\chi}{\chi-1}}.
\end{aligned}$$

B Social Planner Problem

B.1 First best solution if planner chooses task-level labor allocation

$$\max_{\{l_j^n, l_j^g, J \in [N-1,N]\}} \ln Y + \eta \ln\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right) + \psi \int_{J}^{N} l_j^g \, dj$$
$$\text{s.t. } Y = \left(\int_{N-1}^{J} (\gamma_j^n l_j^n)^{\frac{\chi-1}{\chi}} \, dj + \int_{J}^{N} (\gamma_j^g l_j^g)^{\frac{\chi-1}{\chi}} \, dj\right)^{\frac{\chi}{\chi-1}}, \quad l_j^n \geq 0, \quad l_j^g \geq 0.$$

Taking first order conditions, I get:

- $\{l_j^n\} : Y^{\frac{1-\chi}{\chi}} \cdot (\gamma_j^n l_j^n)^{\frac{-1}{\chi}} \gamma_j^n - \frac{\eta}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)} + \mu^n = 0$
- $\{l_j^g\} : Y^{\frac{1-\chi}{\chi}} \cdot (\gamma_j^g l_j^g)^{\frac{-1}{\chi}} \gamma_j^g + \psi - \frac{\eta}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)} + \mu^g = 0$
- $\{J\} : Y^{\frac{1-\chi}{\chi}} \cdot [(\gamma_J^n l_J^n)^{\frac{\chi-1}{\chi}} - (\gamma_J^g l_J^g)^{\frac{\chi-1}{\chi}}] + \frac{\eta(l_J^g - l_J^n)}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)} - \psi l_J^g = 0$, by Leibniz's rule

Using the third FOC,

$$Y^{\frac{1-\chi}{\chi}} \cdot [(\gamma_J^n l_J^n)^{\frac{\chi-1}{\chi}} - (\gamma_J^g l_J^g)^{\frac{\chi-1}{\chi}}] + \frac{\eta(l_J^g - l_J^n)}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)} = \psi l_J^g$$
$$\implies (\gamma_J^n l_J^n)^{\frac{\chi-1}{\chi}} - (\gamma_J^g l_J^g)^{\frac{\chi-1}{\chi}} = \frac{1}{Y^{\frac{1-\chi}{\chi}}} \cdot \left(\psi l_J^g - \frac{\eta(l_J^g - l_J^n)}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)}\right)$$
$$\implies 1 - \left(\frac{\gamma_J^g l_J^g}{\gamma_J^n l_J^n}\right)^{\frac{\chi-1}{\chi}} = \frac{1}{\left(\frac{\gamma_J^n l_J^n}{Y}\right)^{\frac{1-\chi}{\chi}}} \cdot \left(\psi l_J^g - \frac{\eta(l_J^g - l_J^n)}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)}\right)$$
$$\implies \left(\frac{\gamma_J^g l_J^g}{\gamma_J^n l_J^n}\right)^{\frac{\chi-1}{\chi}} = 1 - \frac{1}{\left(\frac{\gamma_J^n l_J^n}{Y}\right)^{\frac{1-\chi}{\chi}}} \cdot \left(\psi l_J^g - \frac{\eta(l_J^g - l_J^n)}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)}\right)$$
$$\implies \frac{\gamma_J^g l_J^g}{\gamma_J^n l_J^n} = \left(1 - \frac{1}{\left(\frac{\gamma_J^n l_J^n}{Y}\right)^{\frac{1-\chi}{\chi}}} \cdot \left(\psi l_J^g - \frac{\eta(l_J^g - l_J^n)}{\left(1 - \int_{N-1}^{J} l_j^n \, dj - \int_{J}^{N} l_j^g \, dj\right)}\right)\right)^{\frac{\chi}{\chi-1}}$$
$$\implies \frac{\gamma_{J_{sp}}^g l_{J_{sp}}^g}{\gamma_{J_{sp}}^n l_{J_{sp}}^n} < 1$$

At the competitive equilibrium threshold J_{ce}, $\gamma_{J_{ce}}^g l_{J_{ce}}^g = \gamma_{J_{ce}}^n l_{J_{ce}}^n$ i.e. $\frac{\gamma_{J_{ce}}^g l_{J_{ce}}^g}{\gamma_{J_{ce}}^n l_{J_{ce}}^n} = 1$. Since by assumption $\frac{\gamma_j^g}{\gamma_j^n}$ is increasing in j, this together with the social planner's solution implies $J_{sp} < J_{ce}$.

B.2 First best solution if planner chooses aggregate labor allocation

$$
\begin{aligned}
&\max_{\{L^n, L^g, J \in [N-1,N]\}} \ln Y + \eta \ln(1 - L^n - L^g) + \psi L^g \\
&\text{s.t. } Y = \left(\left(\int_{N-1}^{J} (\gamma_j^n)^{\chi-1}\, dj \right)^{\frac{1}{\chi}} (L^n)^{\frac{\chi-1}{\chi}} + \left(\int_{J}^{N} (\gamma_j^g)^{\chi-1}\, dj \right)^{\frac{1}{\chi}} (L^g)^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}.
\end{aligned}
$$

Taking first order conditions, I get:

- $\{L^n\} : Y^{\frac{1-\chi}{\chi}} \cdot (L^n)^{\frac{-1}{\chi}} \left(\int_{N-1}^{J} (\gamma_j^n)^{\chi-1}\, dj \right)^{\frac{1}{\chi}} = -\frac{\eta}{1-L^n-L^g}$
- $\{L^g\} : Y^{\frac{1-\chi}{\chi}} \cdot (L^g)^{\frac{-1}{\chi}} \left(\int_{J}^{N} (\gamma_j^g)^{\chi-1}\, dj \right)^{\frac{1}{\chi}} + \tau^g = -\frac{\eta}{1-L^n-L^g}$

Equating the two FOCs,

$$
\begin{aligned}
Y^{\frac{1-\chi}{\chi}} \cdot (L^n)^{\frac{-1}{\chi}} \left(\int_{N-1}^{J} (\gamma_j^n)^{\chi-1}\, dj \right)^{\frac{1}{\chi}} &= Y^{\frac{1-\chi}{\chi}} \cdot (L^g)^{\frac{-1}{\chi}} \left(\int_{J}^{N} (\gamma_j^g)^{\chi-1}\, dj \right)^{\frac{1}{\chi}} + \tau^g \\
\implies \left(\frac{L_n}{L_g} \right)^{\frac{1}{\chi}} \left(\frac{\int_{J}^{N} (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^{J} (\gamma_j^n)^{\chi-1}\, dj} \right)^{\frac{1}{\chi}} &= 1 - \frac{\psi}{Y^{\frac{1-\chi}{\chi}}} \left(\frac{L_n}{\int_{N-1}^{J} (\gamma_j^n)^{\chi-1}\, dj} \right)^{\frac{1}{\chi}} \\
&< 1 = \frac{\gamma_J^g}{\gamma_J^n} = \left(\frac{L_n}{L_g} \right)^{\frac{1}{\chi}} \left(\frac{\int_{J_{ce}}^{N} (\gamma_j^g)^{\chi-1}\, dj}{\int_{N-1}^{J_{ce}} (\gamma_j^n)^{\chi-1}\, dj} \right)^{\frac{1}{\chi}}
\end{aligned}
$$

This implies $J_{sp} < J_{ce}$.

B.3 Implementability of task-specific green wage subsidy

The task-specific wage subsidy under consideration is:

$$
\tau_j^g = \begin{cases} 0 & \text{if } j \leq J_{sp} \\ 1 - \frac{Y_{L^n}}{Y_{L^g}} \frac{\gamma^n(j)}{\gamma^g(j)+\psi} \geq 0 & \text{if } j > J_{sp} \end{cases}
$$

For implementability, I need to show for any $J \leq J_{ce}(L^n, L^g)$, the above subsidy generates a CE with factor prices $W^n = Y_{L^n}(L^n, L^g; J)$ and $W^g = Y_{L^g}(L^n, L^g; J)$ where all tasks below J are produced using non-green labor.

I use similar steps to Acemoglu, Manera, and Restrepo (2020b) to show this. With the above subsidy in place, the unit cost of producing task j with green labor becomes:

$$p_j^g = \begin{cases} \frac{W^g}{\gamma_j^g} & \text{if } j > J \\ \frac{(1-\tau^g)W^g}{\gamma_j^g} & \text{if } j \leq J \end{cases}$$

For all $j \in [0, J)$, $J < J_{ce}(L^n, L^g)$, so

$$\frac{\gamma_j^g}{\gamma_j^n} < \frac{\gamma_{J_{ce}}^g}{\gamma_{J_{ce}}^n} = \frac{Y_{L^g}}{Y_{L^n}} \implies p_j^n = \frac{Y_{L^n}}{\gamma_{J_{ce}}^n} < \frac{Y_{L^g}}{\gamma_{J_{ce}}^g} = p_j^g \implies \text{all tasks produced with non-green labor.}$$

For all $j \in [J, 1)$,

$$\frac{\gamma_j^g}{\gamma_j^n} > \frac{\gamma_{J_{ce}}^g}{\gamma_{J_{ce}}^n} = \frac{(1-\tau^g)Y_{L^g}}{Y_{L^n}} \implies p_j^n = \frac{Y_{L^n}}{\gamma_{J_{ce}}^n} > \frac{Y_{L^g}}{\gamma_{J_{ce}}^g} = p_j^g \implies \text{all tasks produced with green-skilled labor.}$$

The market clears as:

$$L^n = \int_{N-1}^{J} l_j^n \, dj = Y \int_{N-1}^{J} \frac{(p_j^n)^{-\chi}}{\gamma_j^n} \, dj = Y W_n^{-\chi} \int_{N-1}^{J} (\gamma_j^n)^{\chi-1} \, dj$$

$$L^g = \int_{J}^{N} l_j^g \, dj = Y \int_{N-1}^{J} \frac{(p_j^g)^{-\chi}}{\gamma_j^g} \, dj = Y W_g^{-\chi} \int_{J}^{N} (\gamma_j^g)^{\chi-1} \, dj$$

Both conditions hold with equation when $W^n = Y_{L^n}$ and $W^g = Y_{L^g}$. This proves the implementation strategy.